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A NOVEL CHANNEL ESTIMATION METHOD: BLENDING CORRELATION AND LEAST-SQUARES BASED APPROACHES

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ABSTRACT

In this paper we introduce a novel channel estimation method which uses symbol rate sampled signals, based on blending the *least squares based channel estimation* and the *correlation based channel estimation* methods. We first overview the shortcomings of the least squares and the correlation based channel estimation algorithm, where a training sequence is utilized in both cases. The performance of the new channel estimation method will be demonstrated, such that the channel estimation will be more robust, and the overall quality of the estimate will be improved by recovering the pulse shape which is naturally embedded in the overall channel impulse response. We will demonstrate how both methods can be combined effectively to minimize the problems associated with the effective channel delay spread being longer than the known training sequence can support.

1. OVERVIEW OF DATA TRANSMISSION MODEL

For the communications systems utilizing periodically transmitted training sequence, *least-squares* (LS) based channel estimation or the *correlation* based channel estimation algorithms have been the most widely used two alternatives. Both methods use a stored copy of the known transmitted training sequence at the receiver. The properties and the length of the training sequence are generally different depending on the particular communication system's standard specifications. In the sequel, although the examples following the derivations of the blended channel estimator will be drawn from the ATSC digital TV 8-VSB system [1], to the best of our knowledge it could be applied with minor modifications to any digital communication system with linear modulation which employs a training sequence.

The baseband symbol rate sampled matched filter output is given by

$$y[n] \equiv y(t)|_{t=nT} = \sum_k I_k h[n-k] + \tilde{\nu}[n]. \quad (1)$$

where

$$I_k = \left\{ \begin{array}{ll} a_k, & 0 \le k \le N-1 \\ d_k, & N \le n \le N'-1, \end{array} \right\} \in \mathcal{A} \equiv \{\alpha_1, \cdots, \alpha_M\} \quad (2)$$

is the M-ary complex valued transmitted sequence, $\mathcal{A} \subset \mathbf{C}^1$, and $\{a_k\}$ denote the first N symbols within a frame of length N' to indicate that they are the known training symbols; $\tilde{\nu}(t)$ denotes the complex noise process after the matched filter; $h(t)$ is the complex valued impulse response of the composite channel including pulse shaping transmit filter $q(t)$, the physical channel impulse response $c(t)$, and the receive filter $q^*(-t)$, and is given by

$$h(t) = p(t) * c(t) = \sum_{k=-K}^{L} c_k p(t-\tau_k). \quad (3)$$

and $p(t) = q(t) * q^*(-t)$ is the convolution of the transmit and receive filters where $q(t)$ has a finite support of $[-T_q/2, T_q/2]$, and the span of the transmit and receive filters, T_q, is integer multiple of the symbol period, T; that is $T_q = N_q T$, $N_q \in \mathbf{Z}^+$. It is assumed that the time-variations of the channel is slow enough that $c(t)$ can be assumed to be a static inter-symbol interference (ISI) channel throughout the training period with the impulse response

$$c(t) = \sum_{k=-K}^{L} c_k \delta(t-\tau_k) \quad (4)$$

for $0 \le t \le NT$, where N is the number of training symbols. The summation limits K and L denote the number of maximum anti-causal and causal multi-path delays respectively. The multi-path delays τ_k are not assumed to be at integer multiples of the sampling period T. Indeed it is one of the main contributions of this work that we show a robust way of recovering the pulse shape back into the composite channel estimate when the multi-path delays are not at the sampling instants.

2. LEAST-SQUARES CHANNEL ESTIMATION

In order to estimate the channel of Equation (3) the LS based channel estimation algorithm assumes that the starting and the ending points of the channel taps are known. It can be assumed that the impulse response of the discrete-time equivalent composite channel $h[n]$ can be written as a finite dimensional vector

$$\mathbf{h} = [h[-N_a], \cdots, h[-1], h[0], h[1], \cdots, h[N_c]]^T \quad (5)$$

where N_a and N_c denote the number of anti-causal and the causal taps of the channel, respectively, and $N_a + N_c + 1$ is the total memory of the channel. Based on Equation (1) and assuming that $N \ge N_a + N_c + 1$, we will write the matched filter output corresponding only to the known training symbols as

$$\mathbf{Y} = \mathbf{A}\mathbf{h} + \tilde{\boldsymbol{\nu}}, \quad (6)$$

$$\mathbf{Y} = [y[N_c], y[N_c+1], \cdots, y[N-1-N_a]]^T, \quad (7)$$

$$\mathbf{A} = \begin{bmatrix} a_{N_c+N_a} & a_{N_c+N_a-1} & \cdots & a_0 \\ a_{N_c+N_a+1} & a_{N_c+N_a} & \cdots & a_1 \\ \vdots & \vdots & \ddots & \vdots \\ a_{N-1} & a_{N-2} & \cdots & a_{N-1-N_a-N_c} \end{bmatrix}, \quad (8)$$

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where $\mathbf{A}$ is the $(N - N_a - N_c) \times (N_a + N_c + 1)$ convolution matrix corresponding only to the known training symbols, and $\tilde{\boldsymbol{\nu}} = [\tilde{\nu}[N_c], \tilde{\nu}[N_c + 1], \cdots, \tilde{\nu}[N-1-N_a]]^T$. As long as the matrix $\mathbf{A}$ is a tall matrix and of full column rank, that is **(i)** $N \geq 2(N_a + N_c) + 1$, and **(ii)** $\text{rank}\{\mathbf{A}\} = N_a + N_c + 1$ then the least squares solution which minimizes the objective function $J_{LS}(\mathbf{h}) = \|\mathbf{Y} - \mathbf{A}\mathbf{h}\|^2$ exists and unique, and is given by

$$\widehat{\mathbf{h}}_{LS} \;=\; (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{Y}. \tag{9}$$

2.1. Problems Associated with LS Based Algorithms

The first and probably the most crucial problem associated with LS based estimation scheme is the uncertainty associated with the actual channel spread; that is neither the actual channel memory nor the actual first and the last channel taps are known by the receiver. In that case, in order to setup the Equations (7-9) some sort of preprocessing is needed such as correlation of the stored training symbols with the received signal, or some pre-assumed starting and ending points for the channel must be used where the total channel spread is not too long to make the $\mathbf{A}$ matrix a wide matrix, rather than a tall one. Even if the actual channel spread is known by the receiver with the actual starting and ending points, LS based channel estimators have still problems that may be insurmountable to overcome. In such a case LS based estimators suffer from the

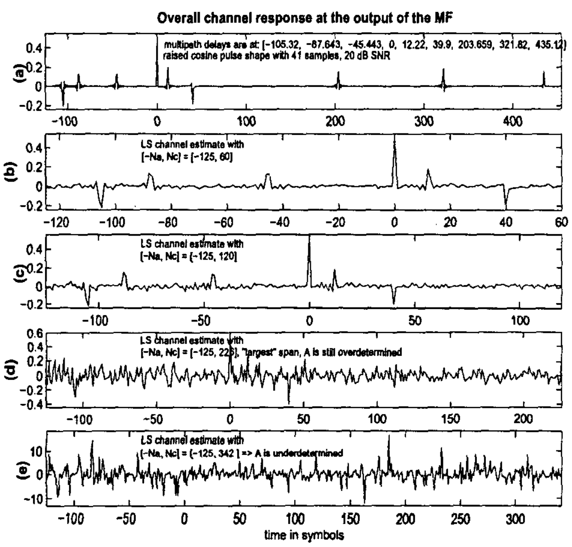

Fig. 1. (a) Shows the real part of a sample channel with a raised cosine pulse shape with 41 samples and 20 dB SNR. Parts (b-e) demonstrate the LS based channel estimation method with $-\tilde{N}_a = -125$; (b) $\tilde{N}_c = 60$; (c) $\tilde{N}_c = 120$; (d) $\tilde{N}_c = 206$. This is the largest span that the receiver can construct with $\mathbf{A}$ matrix being over-determined, and $\tilde{\mathbf{A}}^H\tilde{\mathbf{A}}$ is non-singular. (e) In this case $-\tilde{N}_a = -125, \tilde{N}_c = 342$, thus $\tilde{\mathbf{A}}^H\tilde{\mathbf{A}}$ is singular.

effect of channel spread being too long for the training sequence to effectively support the channel; that is if $N < 2(N_a + N_c) + 1$ such that the number of training symbols is insufficient to support, or create, an overdetermined convolution matrix $\mathbf{A}$ then we have two cases to consider: **(1)** If the receiver attempts to create the full $\mathbf{A}$ matrix as given by Equation (8) the matrix $(\mathbf{A}^H\mathbf{A})$ becomes singular. **(2)** If the receiver does *not* attempt to construct the full $\mathbf{A}$ matrix, instead it constructs an alternative thinner matrix $\tilde{\mathbf{A}}$ with $N \geq 2(\tilde{N}_a + \tilde{N}_c) + 1$ and $\tilde{N}_a < N_a$ and/or $\tilde{N}_c < N_c$ then the inverse of $(\tilde{\mathbf{A}}^H\tilde{\mathbf{A}})$ exists, but the new least squares solution $(\tilde{\mathbf{A}}^H\tilde{\mathbf{A}})^{-1}\tilde{\mathbf{A}}^H\mathbf{Y}$ may no longer resemble the actual channel.

The real part of the channel impulse response seen at the output of the matched filter is shown in Figure 1(a) with multipath delays of [-105.32, -87.643, -45.443, 0, 12.22, 39.9, 203.659, 321.82, 435.12] with a complex raised cosine pulse shape of 41 samples (a channel spread of 580 symbols including the tails of the pulse shape) The training sequence is the same as in 8-VSB transmission standard [1] of length $N = 704$ symbols. With 704 training symbols we can support a channel with a maximum delay spread of only $N_a + N_c + 1 = 352$ taps including the tails of pulse shape. Thus the actual channel has a much longer delay spread than the standard LS based method can handle. As illustrated in Figure 1(b-c) if the receiver assumes a very short channel spread for the sake of getting a cleaner channel estimate within the assumed channel span, it is very likely that we may be missing very significant early pre-cursor multi-paths, or late post-cursor multi-paths.

3. CORRELATION BASED CHANNEL ESTIMATION

This part of the paper is based on the recent work provided in [2]. The sampled received sequence at the matched filter output is denoted by $y[n]$ given by Equation (1), where the precursor symbols at positions $n \geq 0$, post-cursor symbols at positions $n < 0$. The known training sequence will be denoted by $s[n] = a_n$ for $n = 0, \ldots, N-1$. We are assuming that in order to be able to use correlation based channel estimation schemes the training sequences must belong to certain classes of sequences, and thereby possess some certain "nice" correlation properties. One of these classes of sequences is *maximal length pseudo-noise* (PN) sequences. We will denote a PN-sequence of length n as PN_n, and the periodic autocorrelation of a binary valued ($\{+A, -A\}$) PN_n sequence is given by

$$r_{PN_n}[m] \;=\; \begin{cases} A^2 n, & \text{if } m = 0, \pm n, \pm 2n, \cdots \\ -A^2. & \text{otherwise.} \end{cases} \tag{10}$$

Based on the auto-correlation property of Equation (10), we can simply estimate the channel by cross-correlating $s[n]$ (known and stored at the receiver) with the received sequence $y[n]$. The initial pre-cursor and post-cursor impulse response estimate $\tilde{h}_a[n]$ and $\tilde{h}_c[n]$ are determined by

$$\tilde{h}_a[n] \;=\; \sum_{k=0}^{N-1} s^*[k]y[k+n],\; n=0,-1,\cdots,-N+1, \tag{11}$$

$$\tilde{h}_c[n] \;=\; \sum_{k=0}^{N-1} s^*[k]y[k+n],\; n=1,\cdots,N-1. \tag{12}$$

The correlations of Equations (11,12) will clearly yield *peaks* at $\{D_k^a \in \mathbf{Z}^+\}$, for $k = -K, \cdots, -1, 0$, and at $\{D_k^c \in \mathbf{Z}^+\}$, for $k = 1, \cdots, L$. These delays are the sampling instants closest to the locations of the actual physical channel multi-path delays τ_k, $k = -K, \cdots, -1, 0, 1, \cdots, L$, within a symbol interval. Assuming that there is a *level thresholding* algorithm taking place right after the cross-correlations, which is in the form of setting the estimated channel taps to zero if they are below a certain threshold; that is

$$\text{set} \;\; \widehat{h}[n] \;=\; \begin{cases} 0, & \text{if } \tilde{h}[n] < \varepsilon \\ \tilde{h}[n], & \text{otherwise,} \end{cases} \tag{13}$$

for $n = -N+1, \cdots, -1, 0, 1, \cdots, N-1$, then in general we can write the anti-causal and the causal parts of the estimated transfer function *after thresholding* takes place

$$\widehat{H}_a(z) = \tilde{\alpha}_0 \left(\beta_K z^{D_K^a} + \cdots + \beta_1 z^{D_1^a} + 1\right) \quad (14)$$

$$\widehat{H}_c(z) = \tilde{\alpha}_0 \left(\alpha_1 z^{-D_1^c} + \cdots + \alpha_L z^{-D_L^c}\right). \quad (15)$$

where $\widehat{H}(z) = \widehat{H}_a(z) + \widehat{H}_c(z)$, β_k's and α_i's are the complex gains and D_k^a's, D_i^c's are the delays of the estimated channel corresponding to the *pre-cursor* (anti-causal) part, and the *post-cursor* (causal) part respectively. It is assumed that $1 \le D_1^c < \cdots < D_L^c$, and similarly $1 \le D_1^a < \cdots < D_K^a$. Also by the construction of the channel estimates in Equations (11,12) we will have $D_K^a \le N-1$ and $D_L^c \le N-1$.

3.1. Problems Associated with Correlation Based Algorithms

If the PN sequence used is finite and the standard linear correlation is used, then the auto-correlation values corresponding to the non-zero lags will not be constant and will not be as low as $-A^2$. The correlations of the received signal with the stored sequence will be "noisy" because **(i)** the PN sequences are finite in length, they won't achieve their low correlation value for non-zero lag; **(ii)** the span of the cross-correlation includes the known training sequence as well as the random data symbols $\{d_k\}$. Due to these reasons the initial estimate that is obtained out of the correlation of the matched filter output with the stored training sequence will not yield clean and very accurate channel estimate: **(i)** Even if the channel tap estimates $\{\alpha_k, \beta_k\}$, are scaled properly, they will not be very close to the actual channel tap values $\{c_k p(0)\}$ due to the reasons itemized above, as well as due to the fact that the tap values may also be sitting on the tails of the adjacent multi-paths; **(ii)** after the thresholding takes place we will most likely end up only with the peaks of the channel taps, which implies the fact that the tails of the pulse shape that are buried under the "noisy" correlation output may end up being zeroed out entirely. Figure 2(c-d) partly illustrates these problems. This may lead to critical performance loss and slower convergence of the adaptive equalizer that follows the channel estimator.

4. BLENDING CORRELATION AND LS BASED METHODS FOR CHANNEL ESTIMATION

Our blended estimation algorithm will start from the exact same steps of the correlation based estimation scheme outlined by Equations (11,12) and then the thresholding step as in (13) to clean-up the "noise" in the channel estimate. After these steps we will end up with the "cleaned" channel estimate of $\widehat{H}(z)$ given by Equation (14,15), or equivalently we can represent the "cleaned" channel impulse response estimate as $\widehat{\mathbf{h}} = [\widehat{\mathbf{h}}_a^T, \widehat{\mathbf{h}}_c^T]^T$ where

$$\widehat{\mathbf{h}}_a = \tilde{\alpha}_0[\beta_K, \underbrace{0, \cdots, 0}_{D_K^a - D_{K-1}^a - 1 \text{zeros}}, \beta_{K-1}, \cdots, \beta_1, \underbrace{0, \cdots, 0}_{D_1^a - 1 \text{ zeros}}, 1]^T \quad (16)$$

$$\widehat{\mathbf{h}}_c = \tilde{\alpha}_0[\underbrace{0, \cdots, 0}_{D_1^c - 1 \text{ zeros}}, \alpha_1, \cdots, \alpha_{L-1}, \underbrace{0, \cdots, 0}_{D_L^c - D_{L-1}^c - 1 \text{ zeros}}, \alpha_L]^T. \quad (17)$$

Recall that Equation (3) denotes the composite pulse shape convolved by the physical channel impulse response. The idea is that for every multi-path we would like to approximate the shifted and scaled copies of the pulse shape $p(t)$ (shifted by τ_k and scaled by c_k) by a linear combination of three pulse shape functions shifted by half a symbol interval. More precisely

$$c_k p(nT - \tau_k) \approx \begin{cases} \sum_{l=-1}^{1} \gamma_l^{(k)} p((n + D_k^a - \frac{l}{2})T), -K \le k \le -1 \\ \gamma_0^k p(nT), \quad k = 0 \\ \sum_{l=-1}^{1} \gamma_l^{(k)} p((n - D_k^c - \frac{l}{2})T), 1 \le k \le L \end{cases} \quad (18)$$

where $\{\gamma_l^{(k)}, -K \le k \le L\}_{l=-1}^{1} \subset \mathbf{C}^1$. By doing this approximation we can also efficiently recover the tails of the complex pulse shape $p(t)$ which are generally buried under the "noisy" output of the correlation processing, and are completely lost when thresholding is applied. To accomplish this approximation we introduce the vectors $\mathbf{p}_k$, $k = -1, 0, 1$, each containing T spaced samples of the complex pulse shape $p(t)$ shifted by $kT/2$, and

$$\mathbf{p}_k = [p(-N_q T + \frac{kT}{2}), \cdots, p(\frac{kT}{2}), \cdots, p(N_q T + \frac{kT}{2})]^T \quad (19)$$

for $k = -1, 0, 1$, and by concatenating these vectors side by side we define a $(2N_q + 1) \times 3$ matrix $\mathbf{P}$ by

$$\mathbf{P} = [\mathbf{p}_{-1}, \ \mathbf{p}_0, \ \mathbf{p}_1]. \quad (20)$$

Then we form the matrix denoted by Γ whose columns are composed of the shifted vectors $\mathbf{p}_k$, where the shifts represent the relative delays of the multi-paths; that is

$$\Gamma = \begin{bmatrix} \mathbf{P} & & & \\ \mathbf{0}_{(D_K^a + D_L^c) \times 3} & \ddots & \mathbf{0}_{D_K^a \times 1} & \\ & & \mathbf{p}_0 & \\ & & \mathbf{0}_{D_L^c \times 1} & \ddots & \mathbf{0}_{(D_K^a + D_L^c) \times 3} \\ & & & & \mathbf{P} \end{bmatrix} \quad (21)$$

where Γ is of dimension $(D_K^a + D_L^c + 2N_q + 1) \times (3(K+L)+1)$, and $\mathbf{0}_{m \times n}$ denotes an m by n zero matrix. Since it was assumed that $q(t)$ spans N_q symbol durations, it implies that $q[n]$ has $N_q + 1$ sample points, which in turn implies $p[n]$ has $2N_q + 1$ samples. Hence $N_a = D_K^a + N_q$, and $N_c = D_L^c + N_q$. Defining $\gamma^{(k)} = [\gamma_{-1}^{(k)}, \gamma_0^{(k)}, \gamma_1^{(k)}]$ for $-K \le k \le L$, and

$$\gamma = [\gamma^{(-K)}, \cdots, \gamma^{(-1)}, \gamma_0^{(0)}, \gamma^{(1)}, \cdots, \gamma^{(L)}]^T, \quad (22)$$

as the unknown vector of the coefficients $\{\gamma_n^k, n = -1, 0, 1, k = -K, \cdots, 0, \cdots, L\}$, of length $3(K+L)+1$. Then referring to Equations (7, 8), the observation vector $\mathbf{Y}$ is given by

$$\mathbf{Y} = \mathbf{A}\Gamma\gamma + \boldsymbol{\nu} \quad (23)$$

where $\boldsymbol{\nu}$ is the observation noise vector. Using the least squares arguments again, the unknown coefficient vector γ is given by

$$\widehat{\gamma}_{LS} = \left(\Gamma^H \mathbf{A}^H \mathbf{A} \Gamma\right)^{-1} \Gamma^H \mathbf{A}^H \mathbf{Y}. \quad (24)$$

Once the vector $\widehat{\gamma}_{LS}$ is obtained, the new channel estimate $\widehat{\mathbf{h}}_{new}$, with recovered pulse tails, can simply be obtained by

$$\widehat{\mathbf{h}}_{new} = \Gamma \widehat{\gamma}_{LS} \quad (25)$$

In order to have a unique $\widehat{\gamma}_{LS}$ it is required that $\mathbf{A}\Gamma$ be a tall matrix, that is it is required that

$$N - D_K^a - D_L^c - 2N_q \ge 3(K+L) + 1. \quad (26)$$

5. SIMULATIONS OF THE PROPOSED ALGORITHM AND CONCLUSIONS

Figure 2 shows simulation results of the proposed algorithm. A channel with delays at [-105.3, -87.6, -45.4, 0, 12.2, 39.9, 203.6, 321.8, 435.1] with a complex raised cosine pulse shape of 41 samples ($N_q = 20$ and a channel spread of 580 symbols including the tails of the pulse shape) is simulated. The real and imaginary parts of the channel impulse response seen at the output of the matched filter are shown in Figure 2(a-b). The training sequence is the same as in 8-VSB transmission standard [1] of length $N = 704$ symbols. The real and imaginary parts of the correlation processing output before thresholding are shown in Figure 2(c-d). SNR level is 20 dB measured at the output of the matched filter. Apparently the raw channel estimation obtained by correlation processing provides significant peaks corresponding to the dominant channel taps. However the multi-paths which are relatively smaller in magnitude, and pulse shape tails corresponding to almost all multipaths can be buried under the "noise"-like correlation output, and are subject to being completely lost after the thresholding is applied.

Assuming that the thresholding step provides *perfect locations* of the channel taps then the corresponding real and imaginary parts of the channel estimate is given in Figure 2(e-f). Note that the pulse tails are also successfully recovered. Figure 2(g-h) show the real and imaginary parts of the channel estimate with new method when some multi-path delays with smaller magnitudes were not detected by the thresholding step (taps at delays -88 and 435), and when some larger spikes of the correlation output or some larger pulse tails were erroneously picked up as fake channel taps (spikes at 183, and a large pulse tail at 39). Indeed our experience has taught us that this particular scenario can be commonly encountered in practice; the thresholding step of the algorithm may incorrectly pick extra spikes corresponding to correlation output or large pulse tails, or the thresholding step may not detect the presence of the multipaths which are relatively small in magnitude. This particular example demonstrates the robustness and the performance of the proposed method. Missing a few taps or picking up a few extra spikes after the thresholding do not harm the recovered pulse shapes at other multipath locations.

We also note that with 704 training symbols the standard least squares based method can support a channel with a maximum delay spread of only $N_a+N_c+1 = 352$ taps including the tails of pulse shape. The actual channel that we simulate has a much longer delay spread than the standard LS based method can handle. On the other hand, for the channel span of example in Figure 2 the proposed algorithm would still work for the total number of taps of $K + L + 1 \leq \lfloor \frac{704-580-1}{3} \rfloor + 1 = 42$.

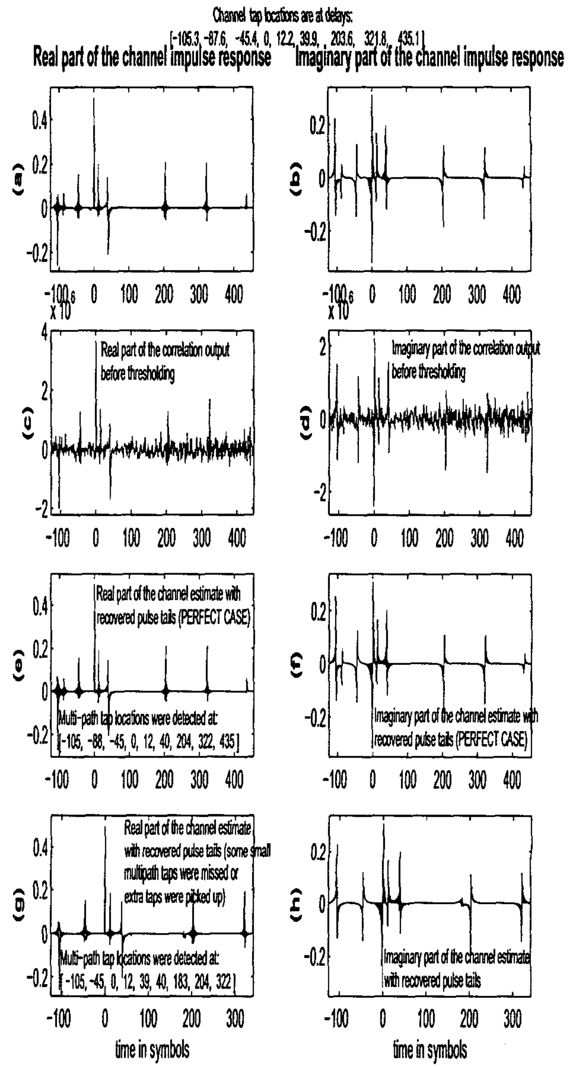

Fig. 2. (a) and (b) show the real and imaginary parts of the channel impulse response seen at the output of the matched filter, with gains $[-0.62, 0.33 - j0.12, 0.73, 1, 0.45, -0.35 + j0.46, 0.54, 0.464 + j0.11, 0.132]$. (c) and (d) show the real and imaginary parts of the correlation processing output. (e) and (f) show the real and imaginary parts of the channel estimate with new method when the multi-path delays were correctly detected by the thresholding part. (g) and (h) show the real and imaginary parts of the channel estimate with new method when some smaller multi-path delays were not detected by the thresholding step (taps at delays -88 and 435), and when some larger spikes of the correlation output or some larger pulse tails were erroneously picked up as fake channel taps (spikes at 183, and pulse tail at 39).

6. REFERENCES

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