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Using the Result of 8-VSB Training Sequence Correlation as a Channel Estimate for DFE Tap Initialization

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Abstract

In this paper we introduce the idea of using 8-VSB [1] training sequence correlation, which is already available in the current digital TV receivers, to initialize the taps of the decision feedback equalizer (DFE). Currently the feed-forward taps of the DFE are initialized to all zeros, with the cursor position initialized to 1. However it is desirable to speed up the convergence of the adaptive equalizer as well as to enable convergence in cases where it would not otherwise occur. Thus we propose to use the 8-VSB training sequence correlation to obtain a multi-path channel estimate, and then to initialize the feed-forward and the feed-back taps of the DFE accordingly.

1 Introduction

DFE's have been widely used in digital receivers due to their implementation simplicity and reasonably good performance under certain multi-path conditions. However, it is desired to enhance the adaptive equalizer training capability of 8-VSB receivers [1]. This may be done by taking advantage of the frame synchronization/training sequence correlation processing that already exists in the receiver for the purpose of data frame synchronization. The output of this processing can be viewed as an estimate of the channel impulse response with the cursor represented by the largest peak. This estimate may be used as a starting point for calculating initial tap values for a DFE.

2 Decision Feedback Equalizer Model

The DFE under consideration is shown in Figure 1. The cursor position is fixed as the last bin of the feed-forward filter, $\tilde{d}_k$ denote soft output of the DFE to the channel decoder, and $\hat{d}_k^-$ denote the past decisions. We will assume that there are $N_{ff}+1$ feed-forward taps including the cursor, and N_{fb} feedback taps. Here $h_{ff}[n]$ and $h_{fb}[n]$ denote the impulse responses of the filters corresponding to the feed-forward and the feedback filter portions of the DFE, respectively.

3 Discrete-time Channel Model, and Channel Estimation

The received sequence is denoted by $u[k]$ with first training symbol at $u[0]$, precursor symbols at positions $k \geq 0$, post-cursor symbols at positions $k < 0$. The known training sequence is $s[k]$ for

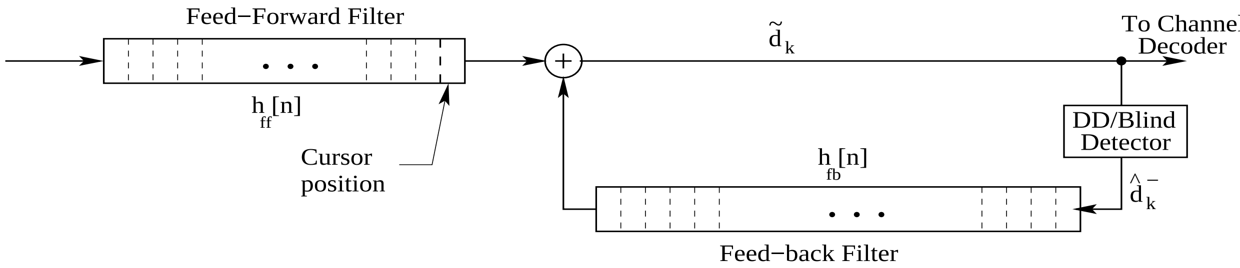

Figure 1: DFE Block Diagram

$k = 0, \ldots, L_{tr} - 1$. Defining the auto and cross-correlation functions, $r_{ss}[m]$ and $r_{su}[m]$ by

$$r_s[m] = \sum_{k=0}^{L_{tr}-1} s[k]s[k+m] \tag{1}$$

$$r_{su}[m] = \sum_{k=0}^{L_{tr}-1} s[k]u[k+m], \tag{2}$$

where $\{s[k] \in \mathcal{A} = \{-A, +A\},\ A \in \mathbf{R}^1\}$ is the training sequence (known and stored at the receiver).

3.1 Correlation Properties of the Training Sequence

We will first go over the correlation properties of a maximal length PN-sequence. We will denote a PN-sequence of length L as PN_L. In general, the periodic autocorrelation of a binary valued $(\{+A, -A\})$ PN_L sequence is given by

$$r_{PN_L}[m] = \begin{cases} LA^2, & \text{if } m = 0, \pm L, \pm 2L, \cdots \\ -A^2. & \text{otherwise.} \end{cases} \tag{3}$$

However if the PN sequence used is finite and the standard linear correlation is used, then the autocorrelation values corresponding to the non-zero lags will not constant and will not be as low as $-A^2$. As a simple illustration consider a sequence composed of six PN_{511} appended back to back, that is let

$$\mathbf{y} = [PN_{511}, PN_{511}, PN_{511}, PN_{511}, PN_{511}, PN_{511}]^T. \tag{4}$$

Then $r_{\mathbf{xy}}[m]$, with $\mathbf{x} = [PN_{511}]^T$, will be given as in Figure 2. It is important to note that we will obtain a low correlation value of $-A^2$ for lags that are not multiples of $L = 511$, corresponding to the intermediate PN_{511} portions of the long sequence $\mathbf{y}$. However as illustrated in Figure 2, for outer most lags we will not achieve this constant and low correlation value; instead we will have a "noise" like correlation due to the finiteness of the sequences.

The training sequence used at the transmitter is, a part of the digital TV standard [1], which is actually

$$\tilde{\mathbf{s}} = [PN_{511}, PN_{63}, \pm PN_{63}, PN_{63}]^T.$$

We also have reserved frame bits and information bits right before and after the training sequence $\tilde{\mathbf{s}}$. As a summary the correlations of the received signal with the stored sequence will be "noisy" because

- the PN sequences are finite in length, they won't achieve their low correlation value for non-zero lag;

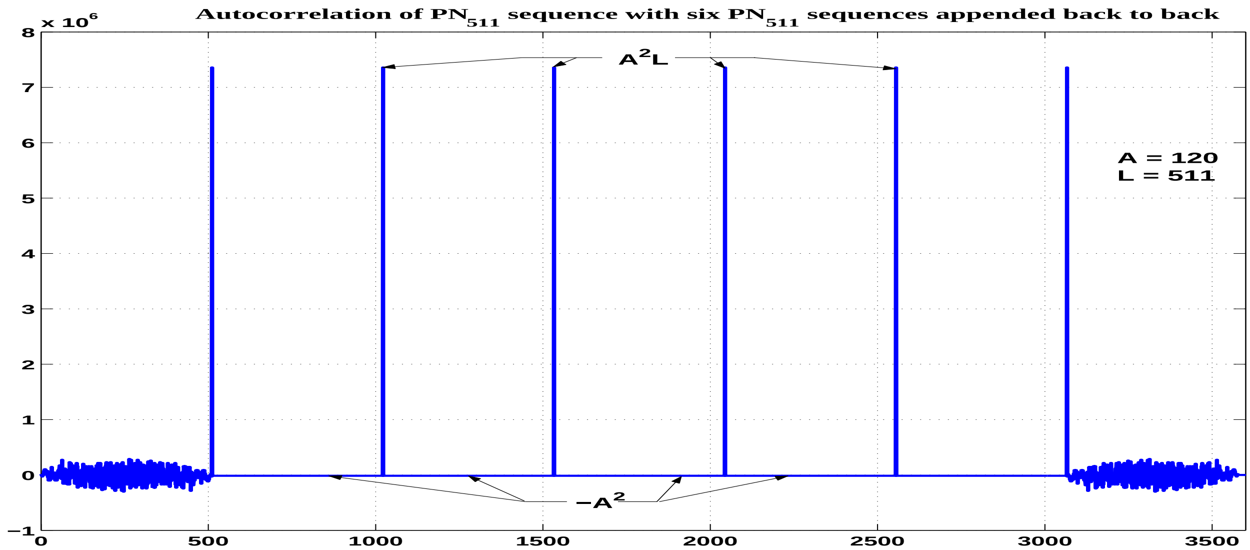

Figure 2: Correlation properties of finite PN sequences. Note the "noisy" correlation at both ends of the correlation values.

- the span of the cross-correlation includes the known training sequence as well as the random data symbols and reserved data symbols.

For the time being we are still working on which sequence to use at the receiver, namely we can use PN_{511} only or the exact same transmitted sequence $\tilde{\mathbf{s}}$.

3.2 Channel Estimation

Based on the auto-correlation property of Equation 3, our claim is that we can simply estimate the channel by cross-correlating $s[k]$ (known and stored at the receiver) with the received sequence $u[k]$

The pre-cursor impulse response estimate $\tilde{h}_a[n]$ is determined from the cross-correlation of the stored training sequence in the receiver and the actual received symbols at lags from 0 to $-N_{ff}$, with respect to the start of the received training sequence. That is

$$\tilde{h}_a[n] = \sum_{k=0}^{L_{tr}-1} s[k]u[k+n] \quad \text{for lags } n = 0, -1, \ldots, -N_{ff}, \tag{5}$$

and similarly post-cursor response $\tilde{h}_c[n]$ is determined from the cross-correlation of the stored training sequence in the receiver and the actual received symbols at lags from 1 to N_{fb}, with respect to the start of the received training sequence which is written compactly as

$$\tilde{h}_c[n] = \sum_{k=0}^{L_{tr}-1} s[k]u[k+n] \quad \text{for lags } n = 1, \ldots, N_{fb}, \tag{6}$$

Let $\tilde{H}(z)$ denote the estimated channel transfer function. Then with respect to the channel estimation procedure outlined by Equations (5,6), $\tilde{H}(z)$ is given by the $\mathcal{Z}$-transform of the *concatenated* channel estimates as

$$\tilde{H}(z) = \sum_{k=-N_{ff}}^{0} \tilde{h}_a[k]z^{-k} + \sum_{k=1}^{N_{fb}} \tilde{h}_c[k]z^{-k}$$

$$= \sum_{k=-N_{ff}}^{N_{fb}} \tilde{h}[k] z^{-k} \quad (7)$$

where the concatenated estimated channel impulse response $\tilde{h}[n]$ is given as

$$\tilde{h}[n] = \begin{cases} \tilde{h}_a[n], & \text{for } -N_{ff} \leq n \leq 0 \\ \tilde{h}_c[n], & \text{for } 1 \leq n \leq N_{fb}. \end{cases} \quad (8)$$

Assuming that there is a *level thresholding* algorithm taking place right after the cross-correlations, we can write $\tilde{H}(z)$ in general as

$$\begin{aligned} \tilde{H}(z) &= \tilde{\beta}_M z^{D^a_M} + \cdots + \tilde{\beta}_2 z^{D^a_2} + \tilde{\beta}_1 z^{D^a_1} + \tilde{\alpha}_0 + \tilde{\alpha}_1 z^{-D_1} + \tilde{\alpha}_2 z^{-D_2} + \cdots + \tilde{\alpha}_N z^{-D_N} \\ &= \tilde{\alpha}_0 \left(\beta_M z^{D^a_M} + \cdots + \beta_1 z^{D^a_1} + 1 + \alpha_1 z^{-D_1} + \cdots + \alpha_N z^{-D_N} \right) \end{aligned} \quad (9)$$

where we defined $\alpha_i = \tilde{\alpha}_i / \tilde{\alpha}_0$, for $1 \leq i \leq N$, and $\beta_k = \tilde{\beta}_k / \tilde{\alpha}_0$, for $1 \leq k \leq M$. Thresholding algorithm will be in the form of setting the estimated channel taps to zero if they are below a certain threshold; that is

$$\text{Set} \quad \tilde{h}[n] = 0, \quad \text{if} \quad \tilde{h}[n] < \varepsilon \quad \text{for } n = -N_{ff}, \cdots, -1, 0, 1, \cdots, N_{fb}. \quad (10)$$

The adoption of the notation used in Equation (9) is based on the fact that, after the thresholding, there are usually very few dominant taps, or equivalently we can say that the channels we encounter are generally *sparse*. A particular *threshold selection/optimization criterion* is to be determined later.

Note that in Equation (9) β_k's are the coefficients of the estimated channel corresponding to the *pre-cursor* (anti-causal) part, and α_i's are the coefficients corresponding to the *post-cursor* (causal) part. D^a_k's, D_i's denote the delays associated with the anti-causal and causal part respectively. As a convention we are assuming that $1 \leq D_1 < D_2 < \cdots < D_N$, and similarly $1 \leq D^a_1 < D^a_2 < \cdots < D^a_M$. Also by the construction of the channel estimates in Equations (5,6) we will have $D^a_M \leq N_{ff}$ and $D_N \leq N_{fb}$.

At this point we can define the causal and anti-causal parts of the transfer function of Equation (9) as

$$H_a(z) = \tilde{\alpha}_0 \left(\beta_M z^{D^a_M} + \cdots + \beta_2 z^{D^a_2} + \beta_1 z^{D^a_1} + 1 \right) \quad (11)$$

$$H_c(z) = \tilde{\alpha}_0 \left(\alpha_1 z^{-D_1} + \alpha_2 z^{-D_2} + \cdots + \alpha_N z^{-D_N} \right) \quad (12)$$

such that $\tilde{H}(z) = H_a(z) + H_c(z)$. The partition of Equations (11,12) will enable us to develop the appropriate *feed-forward* and *feed-back* filter portions of the Decision Feedback Equalizer (DFE).

4 DFE Feed-Forward Filter Tap Initialization

It is known that the feed-forward part of the DFE is supposed to deconvolve the channel output; that is its primary function is to do the *inverse filtering* to get rid of the feed-forward part of the channel, given by Equation (11). Considering the anti-causal part of the channel transfer function given by Equation (11), we need to establish a fast method to give a reasonably accurate initial inverse filter. Let the inverse of the $H_a(z)$ be denoted by $H^I_a(z)$. We would like to have $H_{ff}(z) = H^I_a(z)$ ideally, but since our ideal inverse filters be an infinite impulse response (IIR) filter, and we are going to implement by an finite impulse response (FIR) (a filter with finite number of taps), we would have $H_{ff}(z) \approx H^I_a(z)$. We have 3 different methods to initialize $H^I_a(z)$. However, we must note that the methods 2 and 3 are "zero-forcing" in nature. We will soon modify them to reduce the noise enhancement.

Method 1: The most trivial method is to initialize all of the N_{ff} taps to zero, and let the cursor tap be initialized to 1.

Method 2: This is the Inversion and Truncation (IT) method which determines the linear filter with finite number of taps which is the best inverse of the anti-causal channel to minimize the peak distortion criterion. Recall that

$$\frac{a}{1-r} = a\left(1 + r + r^2 + \cdots\right) = \sum_{k=0}^{\infty} ar^k$$

as long as $|r| < 1$. Using this fact, and since $H_a^I(z) = \frac{1}{H_a(z)}$ we have

$$\begin{aligned} H_a^I(z) &= \frac{1}{\tilde{\alpha}_0}\left(1 - \gamma(z) + \gamma^2(z) - \gamma^3(z) + \cdots\right) \\ &= \frac{1}{\tilde{\alpha}_0}\sum_{k=0}^{\infty}(-1)^k\gamma^k(z) \end{aligned} \tag{13}$$

where

$$\gamma(z) = \beta_M z^{D_M^a} + \cdots + \beta_2 z^{D_2^a} + \beta_1 z^{D_1^a} = \frac{H_a(z)}{\tilde{\alpha}_0} - 1 \tag{14}$$

for $|\gamma(z)| < 1$, where $|\gamma(z)|$ denotes the magnitude of the complex number $\gamma(z)$. The simplest approach here is to initialize the feed-forward filter by the truncated version of Equation (13); that is $H_{ff}(z)$ will be initialized by

$$\begin{aligned} H_{ff}(z) &= \frac{1}{\tilde{\alpha}_0}\left(1 - \gamma(z) + \gamma^2(z) - \gamma^3(z) + \cdots + (-1)^{\tilde{N}_{ff}}\gamma^{\tilde{N}_{ff}}(z)\right) \\ &= \frac{1}{\tilde{\alpha}_0}\sum_{k=0}^{\tilde{N}_{ff}}(-1)^k\gamma^k(z) \end{aligned} \tag{15}$$

where $\tilde{N}_{ff} = \lfloor \frac{N_{ff}}{D_M^a} \rfloor$. Notice that the sum in Equation 15 is finite regardless of $|\gamma(z)|$.

It must be noted that Equation (15) might be *truncated too early* depending on the delays and the length of the feed-forward filter. We can further improve Equation (15) by assuming that we have, for example $M = 2$ dominant multi-paths on the anti-causal (precursor) estimated channel. The results shown here could easily be extended to arbitrary M delays, for all $M \leq N_{ff}$.

We first introduce the peak distortion criterion, $\mathcal{D}_p$, as established by Lucky [2, 3, 4] which is defined as:

$$\mathcal{D}_p = \frac{1}{|q_0|}\sum_{n=1}^{D_M^a+N_{ff}+1} |q_n - \hat{q}_n| \tag{16}$$

where $\hat{\mathbf{q}} = [\hat{q}_0, \hat{q}_1, \cdots, \hat{q}_{D_M^a+N_{ff}+1}]^T$ is the *desired equalized response* out of the feed-forward equalizer (channel and equalizer combined), and $\mathbf{q} = [q_0, q_1, \cdots, q_{D_M^a+N_{ff}+1}]^T$ is the *actual equalized response*. With a zero-forcing equalizer (ZF), the tap coefficients $\mathbf{h}_{ff}$ are chosen to minimize the peak distortion of the equalized channel defined as in Equation (16). For our purposes we particularly choose $\hat{\mathbf{q}} = [1, 0, \cdots, 0]^T$. It was shown that [2, 5] if the initial distortion $\mathcal{D}_{\text{init}}$ without equalization is less than unity, that is

$$\mathcal{D}_{\text{init}} = \frac{1}{|h_0^a|}\sum_{n=1}^{D_M^a} |h_n^a| < 1 \tag{17}$$

then $\mathcal{D}_p$ is minimized by those $N_{ff}+1$ equalizer tap values. If the initial distortion before equalization is greater than unity, the ZF criterion is no longer guaranteed to minimize the peak distortion. However, even if the initial distortion is greater than unity, using the channel estimates to initialize the DFE feed-forward taps as established in the rest of the Method 2, and following Method 3, will still yield a faster convergence as compared to the DFE with taps initialized by Method 1.

Following the developments in [5], for the anti-causal channel estimate of

$$\begin{aligned}\mathbf{h}_a(n) &= [h^a_n, h^a_{n-1}, \cdots, h^a_{n-D^a_M}]^T \\ &= \tilde{\alpha}_0[1, \underbrace{0, \cdots, 0}_{D^a_1-1 \text{ zeros}}, \beta_1, \underbrace{0, \cdots, 0}_{D^a_2-D^a_1-1 \text{ zeros}}, \beta_2, \cdots, \beta_{M-1}, \underbrace{0, \cdots, 0}_{D^a_M-D^a_{M-1}-1 \text{ zeros}}, \beta_M]^T \end{aligned} \tag{18}$$

we form the convolution matrix $\mathbf{H}_{\text{conv}}$ as

$$\mathbf{H}_{\text{conv}} = [\mathbf{h}_a(0), \mathbf{h}_a(1), \cdots, \mathbf{h}_a(N_{ff})], \tag{19}$$

and we introduce the desired response vector truncated to match the equalizer length $\tilde{\mathbf{q}} = [\tilde{q}_0, \tilde{q}_1, \cdots, \tilde{q}_{N_{ff}}]^T$. Then, as long as the inverse of the convolution matrix exists, the unique vector of optimal feed-forward equalizer tap gains, $\mathbf{h}_{ff}$, satisfies

$$\mathbf{h}^T_{ff}\mathbf{H}_{\text{conv}} = \tilde{\mathbf{q}}^T \qquad \Longrightarrow \qquad \mathbf{h}^T_{ff} = \tilde{\mathbf{q}}^T\mathbf{H}^{-1}_{\text{conv}}. \tag{20}$$

Theorem 1: Let the anti-causal channel be given by

$$H_a(z) = \tilde{\alpha}_0\left(1 + \beta_1 z^{D^a_1} + \beta_2 z^{D^a_2}\right) \tag{21}$$

with $1 \le D^a_1 < D^a_2 \le N_{ff}$, or equivalently

$$\mathbf{h}_a = \tilde{\alpha}_0[1, \underbrace{0, \cdots, 0}_{D^a_1-1 \text{ zeros}}, \beta_1, \underbrace{0, \cdots, 0}_{D^a_2-D^a_1-1 \text{ zeros}}, \beta_2]^T, \tag{22}$$

and given that the initial distortion condition of Equation (17) holds; then the transfer function of the unique linear feed-forward equalizer with $N_{ff}+1$ taps which *minimizes the peak distortion criterion* is given by

$$H_{ff}(z) = \frac{1}{\tilde{\alpha}_0}\sum_{k=0}^{k_{max}}\sum_{l=0}^{l_{max}(k)}\binom{k}{l}(-1)^k\left(\beta_1^{k-l}\beta_2^l z^{D_{k,l}}\right) \tag{23}$$

where

$$D_{k,l} = (k-l)D^a_1 + lD^a_2 = kD^a_1 + (D^a_2 - D^a_1)l, \tag{24}$$

$$k_{max} = \left\lfloor \frac{N_{ff}}{D^a_1} \right\rfloor \tag{25}$$

$$l_{max}(k) = \left\lfloor \frac{N_{ff} - kD^a_1}{D^a_2 - D^a_1} \right\rfloor. \tag{26}$$

which also satisfies the Equation (20) where the desired response is given by

$$\tilde{\mathbf{q}} = [1, \underbrace{0, \cdots, 0}_{N_{ff} \text{ zeros}}]^T \tag{27}$$

and the convolution matrix is given by

$$
\mathbf{H}_{\text{conv}} = \frac{1}{\tilde{\alpha}_0}\left[\begin{array}{cccccccccccc}
1 & 0 & \cdots & 0 & \beta_1 & 0 & \cdots & 0 & \beta_2 & 0 & \cdots & \\
0 & 1 & 0 & \cdots & 0 & \beta_1 & 0 & \cdots & 0 & \beta_2 & 0 & \cdots \\
0 & \cdots & \ddots & 0 & \cdots & 0 & \ddots & 0 & \cdots & 0 & \ddots & \cdots \\
0 & \vdots & 0 & \ddots & 0 & \cdots & 0 & \beta_1 & & 0 & \cdots & \beta_2 \\
\vdots & & & & \ddots & & & & \ddots & 0 & \cdots & 0 \\
0 & \vdots & 0 & & 0 & 1 & 0 & & & \ddots & \cdots & \vdots \\
0 & \vdots & 0 & & & 0 & & & & \cdots & & \beta_1 \\
 & & & & & \cdots & & \ddots & & & & 0 \\
\vdots & & & & & \cdots & & & & & & \cdots \\
 & & & & & \cdots & & & & 0 & 1 & 0 \\
0 & & & & & \cdots & & & & & 0 & 1
\end{array}\right]_{(N_{ff}+1)\times(N_{ff}+1)},
$$

Before we give the full proof of the theorem, we will provide the underlying motivation behind it. Since our desired response is given by (27), then by Equation (20) we have

$$
\mathbf{h}_{ff}^T \;=\; [1,\ 0,\ \cdots\ ,\ 0]\mathbf{H}_{\text{conv}}^{-1} \tag{28}
$$

which simply implies that $\mathbf{h}_{ff}^T$ is *the first row of the inverse of* $\mathbf{H}_{\text{conv}}$. Let $[\mathbf{H}]_{\{m,n\}}$ denote the $\{m,n\}$'th element , $[\mathbf{H}]_{\{m,:\}}$ denote the m'th row, and similarly $[\mathbf{H}]_{\{:,n\}}$ denote the n'th column of the matrix $\mathbf{H}$. Let $\text{cof}_{\{m,n\}}\mathbf{H}$ be the cofactor of the matrix $\mathbf{H}$ with respect to the $\{m,n\}$'th element. Then with this notation in mind we have

$$
\begin{aligned}
\mathbf{h}_{ff}^T &= \left[\mathbf{H}_{\text{conv}}^{-1}\right]_{\{1,:\}} && (29)\\
&= \frac{1}{\det\left(\mathbf{H}_{\text{conv}}\right)}\left[\text{adj}\left(\mathbf{H}_{\text{conv}}\right)\right]_{\{1,:\}} && (30)\\
&= \frac{1}{\tilde{\alpha}_0^{N_{ff}+1}}\left[\text{cof}_{\{1,1\}}\mathbf{H}_{\text{conv}}, \text{cof}_{\{2,1\}}\mathbf{H}_{\text{conv}}, \cdots, \text{cof}_{\{N_{ff}+1,1\}}\mathbf{H}_{\text{conv}}\right]. && (31)
\end{aligned}
$$

Equation (30) is simply obtained from Equation (29) by using the definition of the matrix inverse, and Equation (31) is obtained from Equation (30) using the definition of adjoint matrix which is the transpose of the cofactor matrix, and the $\{m,n\}$'th element of the adjoint matrix is given by

$$
\left[\text{adj}\left(\mathbf{H}\right)\right]_{\{m,n\}} \;=\; \text{cof}_{\{n,m\}}\mathbf{H}. \tag{32}
$$

The important argument here we make is that Equation (23) introduced in Theorem gives us the same result with Equations (29-31) for the channel of (21), or equivalently for the channel of (22).

Proof: In order to prove this theorem, it is sufficient to make the observation that Equation (23) is nothing but the improved truncated expansion of Equation (15) with the number of terms included in the truncated expansion is determined by the upper limits of the outer summation k_{max} and inner summation $l_{max}(k)$ respectively. It is also required to prove the following lemma on the ordering of the exponents of z before deriving the equations governing k_{max} and $l_{max}(k)$.

Lemma: For $D_1^a < D_2^a$ we have $D_{k,(l+1)} > D_{k,l}$.

Proof:

$$D_{k,(l+1)} = (k-(l+1))\,D_1^a + (l+1)D_2^a = kD_1^a + (D_2^a - D_1^a)\,l. \tag{33}$$

Since $D_{k,l} = kD_1^a + (D_2^a - D_1^a)l$, and $D_1^a < D_2^a$ we have

$$D_{k,(l+1)} - D_{k,l} = D_2^a - D_1^a > 0$$

Hence $D_{k,(l+1)} > D_{k,l}$. $\square_{\text{lemma}}$

Lemma shows the ascending ordering of the exponents of the z for each k. The upper limit for the outer summation on k is found by observing that the maximum allowable k_{max} times the smallest delay, D_1^a must be less than equal to N_{ff}, that is $k_{max}D_1^a \le N_{ff}$, and this gives us $k_{max} = \lfloor \frac{N_{ff}}{D_1^a} \rfloor$. Similarly, for each k, the upper limit on the inner summation over l is given by $D_{k,l_{max}(k)} \le N_{ff}$; then using Equation (24) we have $kD_1^a + (D_2^a - D_1^a)l_{max}(k) \le N_{ff}$, which gives us for every outer summation index k we have $l_{max}(k) = \left\lfloor \frac{N_{ff}-kD_1^a}{D_2^a-D_1^a} \right\rfloor$. $\square_{\text{Theorem 1}}$

Example: We can consider a simple example where we have two dominant paths at the anti-causal channel response. Assume with $D_1^a = 19$, $D_2^a = 30$ and assume we have a feed-forward filter with $N_{ff}+1 = 100$ taps (including the cursor). Then $\tilde{N}_{ff} = \lfloor \frac{99}{30} \rfloor = 3$, and according to Equation (15) (ignoring $\tilde{\alpha}_0$ term)

$$\begin{aligned} H_{ff}(z) &= \sum_{k=0}^{3} (-1)^k \left(\beta_1 z^{19} + \beta_2 z^{30}\right)^k \\ &= 1 - \left(\beta_1 z^{19} + \beta_2 z^{30}\right) + \left(\beta_1 z^{19} + \beta_2 z^{30}\right)^2 - \left(\beta_1 z^{19} + \beta_2 z^{30}\right)^3 \\ &= 1 - \left(\beta_1 z^{19} + \beta_2 z^{30}\right) + \left(\beta_1^2 z^{38} + 2\beta_1\beta_2 z^{49} + \beta_2^2 z^{60}\right) \\ &\quad - \left(\beta_1^3 z^{57} + 3\beta_1^2\beta_2 z^{68} + 3\beta_1\beta_2^2 z^{79} + \beta_2^3 z^{90}\right) \end{aligned} \tag{34}$$

We can easily argue that the expansion in (34) is truncated too short, and using the formulation developed in Equations (21–26) we can include the terms that are the powers of the smaller delay D_1^a and possibly combinations with D_2^a such that our improved feed-forward filter initialization will start with few more extra taps

$$\begin{aligned} H_{ff}(z) &= 1 - \left(\beta_1 z^{19} + \beta_2 z^{30}\right) + \left(\beta_1^2 z^{38} + 2\beta_1\beta_2 z^{49} + \beta_2^2 z^{60}\right) \\ &\quad - \left(\beta_1^3 z^{57} + 3\beta_1^2\beta_2 z^{68} + 3\beta_1\beta_2^2 z^{79} + \beta_2^3 z^{90}\right) \\ &\quad + \beta_1^4 z^{76} + 4\beta_1^3\beta_2 z^{87} + 6\beta_1^2\beta_2^2 z^{98} + \beta_1^5 z^{95} \end{aligned} \tag{35}$$

Now we can give the generalized version of Theorem 1, where the Equations (23-26) are extended for the anti-causal channel estimate with M taps with the transfer function

$$H_a(z) = \tilde{\alpha}_0 \left(\beta_M z^{D_M^a} + \cdots + \beta_2 z^{D_2^a} + \beta_1 z^{D_1^a} + 1\right), \tag{36}$$

or equivalently with impulse response vector

$$\begin{aligned} \mathbf{h}_a(n) &= [h_n^a, h_{n-1}^a, \cdots, h_{n-D_M^a}^a]^T \\ &= \tilde{\alpha}_0[1, \underbrace{0,\cdots,0}_{D_1^a-1 \text{ zeros}}, \beta_1, \underbrace{0,\cdots,0}_{D_2^a-D_1^a-1 \text{ zeros}}, \beta_2, \cdots, \beta_{M-1}, \underbrace{0,\cdots,0}_{D_M^a-D_{M-1}^a-1 \text{ zeros}}, \beta_M]^T. \end{aligned} \tag{37}$$

Theorem 2: Let the M tap anti-causal channel be given by (36), or (37), with $1 \leq D_1^a < \cdots < D_M^a \leq N_{ff}$, and given that the initial distortion condition of Equation (17) holds; then the transfer function of the unique linear feed-forward equalizer with $N_{ff}+1$ taps which *minimizes the peak distortion criterion* is given by

$$\begin{aligned} H_{ff}(z) &= \frac{1}{\tilde{\alpha}_0} \sum_{k_1=0}^{k_{1,max}} \sum_{k_2=0}^{k_{2,max}(k_1)} \cdots \sum_{k_{M-1}=0}^{k_{M-1,max}(k_1,k_2,\cdots,k_{M-2})} \sum_{k_M=0}^{k_{M,max}(k_1,k_2,\cdots,k_{M-1})} \\ & \qquad \binom{k_1}{k_2}\binom{k_2}{k_3}\cdots\binom{k_{M-1}}{k_M}(-1)^{k_1}\left(\beta_1^{k_1-k_2}\beta_2^{k_2-k_3}\cdots\beta_{M-1}^{k_{M-1}-k_M}\beta_M^{k_M} z^{D(k_1,\cdots,k_M)}\right) \\ &= \frac{1}{\tilde{\alpha}_0} \sum_{k_1=0}^{k_{1,max}} \sum_{k_2=0}^{k_{2,max}(k_1)} \cdots \sum_{k_{M-1}=0}^{k_{M-1,max}(k_1,k_2,\cdots,k_{M-2})} \sum_{k_M=0}^{k_{M,max}(k_1,k_2,\cdots,k_{M-1})} \\ & \qquad \frac{k_1!\,(-1)^{k_1}\,\beta_1^{k_1-k_2}\beta_2^{k_2-k_3}\cdots\beta_{M-1}^{k_{M-1}-k_M}\beta_M^{k_M} z^{D(k_1,\cdots,k_M)}}{k_M!\prod_{l=1}^{M-1}((k_l-k_{l+1})!)} \end{aligned} \tag{38}$$

where

$$\begin{aligned} D(k_1,\cdots,k_M) &= (k_1-k_2)D_1^a + (k_2-k_3)D_2^a + \cdots + (k_{M-1}-k_M)D_{M-1}^a + k_M D_M^a \\ &= k_1 D_1^a + (D_2^a - D_1^a)k_2 + \cdots + (D_M^a - D_{M-1}^a)k_M, \end{aligned} \tag{39}$$

$$k_{1,max} = \left\lfloor \frac{N_{ff}}{D_1^a} \right\rfloor \tag{40}$$

$$k_{n,max}(k_1,\cdots,k_{n-1}) = \left\lfloor \frac{N_{ff} - k_1 D_1^a - \sum_{l=2}^{n-1}(D_l^a - D_{l-1}^a)k_l}{D_n^a - D_{n-1}^a} \right\rfloor, \quad \text{for } 2 \leq n \leq M \tag{41}$$

which also satisfies the Equation (20) where the desired response is given by

$$\tilde{\mathbf{q}} = [1, \underbrace{0,\cdots,0}_{N_{ff}\ \text{zeros}}]^T. \tag{42}$$

Method 3: This method is based on a method solving the Equation (20), $\mathbf{h}_{ff}^T \mathbf{H}_{\text{conv}} = \tilde{\mathbf{q}}^T$ without calculating the inverse of $\mathbf{H}_{\text{conv}}$ explicitly, for

$$\tilde{\mathbf{q}} = [1, \underbrace{0,\cdots,0}_{N_{ff}\ \text{zeros}}]^T \tag{43}$$

and

$$\begin{aligned} \mathbf{h}_a(n) &= [h_n^a, h_{n-1}^a, \cdots, h_{n-N_{ff}}^a]^T \\ \mathbf{H}_{\text{conv}} &= [\mathbf{h}_a(0), \mathbf{h}_a(1), \cdots, \mathbf{h}_a(N_{ff})]. \end{aligned}$$

We will first develop the solution $\mathbf{h}_{ff}^T$ for the general case (channel with possibly all $N_{ff}+1$ non-zero taps), then the solution for the sparse channel case will follow afterwards. In this case our convolution matrix will be an upper triangular Toeplitz matrix, and will be given as

$$\mathbf{H}_{\text{conv}} = \begin{bmatrix} h_0^a & h_1^a & h_2^a & & \cdots & & & h_{N_{ff}-1}^a & h_{N_{ff}}^a \\ 0 & h_0^a & h_1^a & h_2^a & & & & & h_{N_{ff}-1}^a \\ 0 & 0 & h_0^a & h_1^a & h_2^a & & & & \vdots \\ 0 & \vdots & 0 & \ddots & \ddots & \ddots & & & \\ 0 & \vdots & 0 & & \ddots & \ddots & h_2^a & & \\ & & & & & \ddots & h_1^a & h_2^a & \vdots \\ \vdots & & & & & & h_0^a & h_1^a & h_2^a \\ & & & & & & 0 & h_0^a & h_1^a \\ 0 & 0 & & & \cdots & & 0 & 0 & h_0^a \end{bmatrix}_{(N_{ff}+1)\times(N_{ff}+1)},$$

or equivalently the $\{m,n\}$'th element of the convolution matrix $\mathbf{H}_{\text{conv}}$ is given by

$$[\mathbf{H}_{\text{conv}}]_{\{m,n\}} = \begin{cases} h_{n-m}^a, & \text{if } m \leq n \\ 0, & \text{otherwise} \end{cases} \tag{44}$$

for $1 \leq m, n \leq N_{ff}+1$. The required vector

$$\mathbf{h}_{ff} = [h_0^{ff}, h_1^{ff}, \cdots, h_{N_{ff}}^{ff}]^T$$

can be obtained by

$$\begin{aligned} h_0^{ff} &= \frac{1}{h_0^a} \\ h_1^{ff} &= -\frac{1}{h_0^a}\left(h_0^{ff} h_1^a\right) \\ h_2^{ff} &= -\frac{1}{h_0^a}\left(h_0^{ff} h_2^a + h_1^{ff} h_1^a\right) \\ h_3^{ff} &= -\frac{1}{h_0^a}\left(h_0^{ff} h_3^a + h_1^{ff} h_2^a + h_2^{ff} h_1^a\right) \\ \vdots &= \vdots \\ h_{N_{ff}}^{ff} &= -\frac{1}{h_0^a}\left(h_0^{ff} h_{N_{ff}}^a + h_1^{ff} h_{N_{ff}-1}^a + \cdots + h_{N_{ff}-1}^{ff} h_1^a\right) \end{aligned} \tag{45}$$

where the general recursion follows trivially by induction that

$$h_k^{ff} = -\frac{1}{h_0^a} \sum_{n=0}^{k-1} h_n^{ff} h_{k-n}^a, \qquad \text{for } k = 1, 2, \cdots, N_{ff} \tag{46}$$

with the initialization of $h_0^{ff} = 1/h_0^a$. It is important to note that in Equation (46), h_k^{ff} depends only on the set $\{h_0^{ff}, h_1^{ff}, \cdots, h_{k-1}^{ff}\}$ which has been calculated in the previous steps.

The computational complexity of the algorithm (46) has been studied. The number of multiplication operation as a function of number of multi-path components has been given in Figure 3.

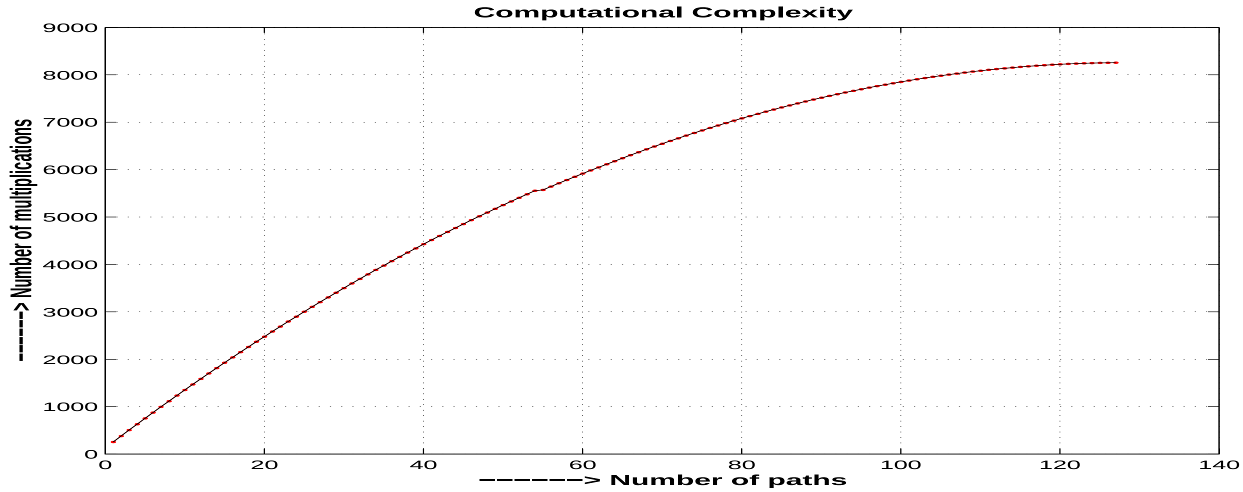

Figure 3: Number of multiplication operations as a function of number of multi-paths for the algorithm of (46).

5 DFE Feedback Filter Tap Initialization

The feedback filter $h_{fb}[n]$ should be initialized to the convolution of the estimated channel impulse response and the feed-forward filter, which is given by

$$h_{fb}[n] = \tilde{h}[n] * h_{ff}[n], \text{ for } 1 \leq n \leq N_{fb} \tag{47}$$

where $*$ denotes the linear convolution operation.

6 Practical Applications

- Use correlation channel estimate for post-cursor only tap initialization (method with the least computation):
  - Initialize all taps to zero, except cursor tap = 1 (Method-1 in previous section)
  - When first training sequence arrives, run equalizer in training mode and simultaneously do correlation channel estimate
  - At end of training sequence, leave precursor taps with trained values, initialize post-cursor taps to convolution of the post-cursor channel estimate and precursor trained values
- Use correlation channel estimate for post-cursor and precursor tap initialization, case A:
  - When the first training sequence arrives it is stored in memory (quantized soft symbols from channel) at the same time the correlation is being calculated.
  - At the end of the sequence, an FIR approximation of the precursor channel inverse is calculated and used to initialize the precursor taps (use methods 2 or 3 established in the previous section).
  - The post-cursor taps are initialized to the convolution of the post-cursor channel estimate and the calculated precursor tap values.

- The training sequence from memory is run through the DFE running in training mode.
- Data symbols received during the steps 2, 3 and 4 above are discarded (at most 1 or 2 segments). At the completion of the 4th step, received data may be run through the DFE in DD/blind mode.

- Use correlation channel estimate for post-cursor and precursor tap initialization, case B:
  - When the first training sequence arrives, the correlation channel estimate is calculated.
  - At the end of the sequence, an FIR approximation of the precursor channel inverse is calculated and used to initialize the precursor taps (use methods 2 or 3 established in the previous section).
  - The post-cursor taps are initialized to the convolution of the post-cursor channel estimate and the calculated precursor tap values.
  - The equalizer is frozen until the next training sequence.
  - When the next training sequence arrives, we start the equalizer in training mode and simultaneously do another correlation/channel estimate. If the estimate of the channel is close enough to the 1st estimate (by some criterion TBD), then we let the equalizer continue. If not, we start over again. This will continue until the channel is sufficiently stationary for one VSB frame time.

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