A NOVEL STRUCTURED CHANNEL ESTIMATION METHOD FOR SPARSE CHANNELS WITH APPLICATIONS TO MULTI-ANTENNA DIGITAL TV RECEIVERS

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ABSTRACT

In this paper we introduce a novel channel impulse response (CIR) estimation method, for sparse multipath channels, with applications to digital TV receivers with multiple antennae. Our new method uses symbol rate samples of the receiver matched filter output, and it is based on blending the *least squares based channel estimation* and the *correlation based channel estimation* methods. We first overview the shortcomings of the least squares and the correlation based channel estimation algorithm, where a training sequence is utilized in both cases. The performance of the new channel estimation method will be demonstrated, such that the channel estimation will be more robust, and the overall quality of the estimate will be improved by recovering the pulse shape which is naturally embedded in the overall channel impulse response. We will demonstrate how both methods can be combined effectively to minimize the problems associated with the effective channel delay spread being longer than the known training sequence can support. Examples are drawn from the ATSC digital TV 8-VSB system [1] with a multi-antenna receiver. The delay spread for digital TV systems can be as long as several hundred times the symbol duration; however digital TV channels are, in general, *sparse* where there are only a few dominant multipaths. Finally we derive the noise variance estimator.

1. OVERVIEW OF DATA TRANSMISSION MODEL

For the communications systems utilizing periodically transmitted training sequence, *least-squares* (LS) based channel estimation or the *correlation* based channel estimation algorithms have been the most widely used two alternatives. Both methods use a stored copy of the known transmitted training sequence at the receiver. The properties and the length of the training sequence are generally different depending on the particular communication system's standard specifications. In the sequel, although the examples following the derivations of the blended channel estimator will be drawn from the ATSC digital TV 8-VSB system [1], to the best of our knowledge it could be applied with minor modifications to any digital communication system with linear modulation which employs a training sequence.

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Let $N_A \geq 1$ denote the number of antennae used at the receiver. Then the baseband symbol rate sampled receive filter output for ith antenna is given by

$$y_i[n] \equiv y_i(t)|_{t=nT} = \sum_k I_k h_i[n-k] + \tilde{\nu}_i[n], \qquad (1)$$

for $1 \leq i \leq N_A$, where

$$I_k = \left\{ \begin{array}{ll} a_k, & 0 \leq k \leq N-1 \\ d_k, & N \leq n \leq N'-1, \end{array} \right\} \in \mathcal{A} \equiv \{\mu_1, \cdots, \mu_M\} \qquad (2)$$

is the M-ary complex valued transmitted sequence from the set $\mathcal{A} \equiv \{\mu_1, \cdots, \mu_M\} \subset \mathbb{C}^1$, and $\{a_k\}$ denote the first N symbols within a frame of length N' to indicate that they are the known training symbols; defining $\nu_i(t)$ as the complex additive white Gaussian noise with variance $N_{0,i}$ per real and imaginary part, $\tilde{\nu}_i(t) = \nu_i(t) * q^*(-t)$ denotes the colored complex noise after the pulse matched filter; $h_i(t)$ is the complex valued impulse response of the composite channel seen by the ith antenna including pulse shaping transmit filter $q(t)$, the physical channel impulse response $c_i(t)$, and the receive filter $q^*(-t)$, and is given by

$$h_i(t) \quad = \quad p(t) * c_i(t) = \sum_{k=-K}^{L} c_{i,k} p(t - \tau_{i,k}), \qquad (3)$$

and $p(t) = q(t) * q^*(-t)$ is the convolution of the transmit and receive filters where $q(t)$ has a finite support of $[-T_q/2, T_q/2]$, and the span of the transmit and receive filters, T_q, is integer multiple of the symbol period, T; that is $T_q = N_q T$, $N_q \in \mathbb{Z}^+$. $\{c_{i,k}\} \subset \mathbb{C}^1$ denote complex valued physical channel gains, and $\{\tau_{i,k}\}$ denote the multipath delays, or the Time-Of-Arrivals (TOA). It is assumed that the time-variations of the channel is slow enough that $c_i(t)$ can be assumed to be a static inter-symbol interference (ISI) channel throughout the training period with the impulse response

$$c_i(t) \quad = \quad \sum_{k=-K}^{L} c_{i,k} \delta(t - \tau_{i,k}) \qquad (4)$$

for $0 \leq t \leq NT$, where N is the number of training symbols. The summation limits K and L denote the number of maximum anti-causal and causal multi-path delays respectively. The multipath delays $\tau_{i,k}$ are *not* assumed to be at integer multiples of the sampling period T. Indeed it is one of the main contributions of this work that we show a robust way of recovering the pulse shape back into the composite channel estimate when the multi-path delays are not at the symbol-spaced sampling instants.

1.1. Receiver Antenna Array Geometry

For a receiver employing an antenna array we will make the basic assumption that every multipath reflection reaches all the antenna elements at the same time but with different phase angles. This assumption enables us to drop the antenna index from the TOA's; that is in the sequel we will use $\tau_{i,k} = \tau_k$ for $-K \leq k \leq L$.

In the sequel we provide a brief derivation of the phase difference in the multipath components at each antenna element. Considering Figure 1 it is straightforward to show that the distance d that the plane wave which is on the second antenna has to travel to reach the first antenna is given by

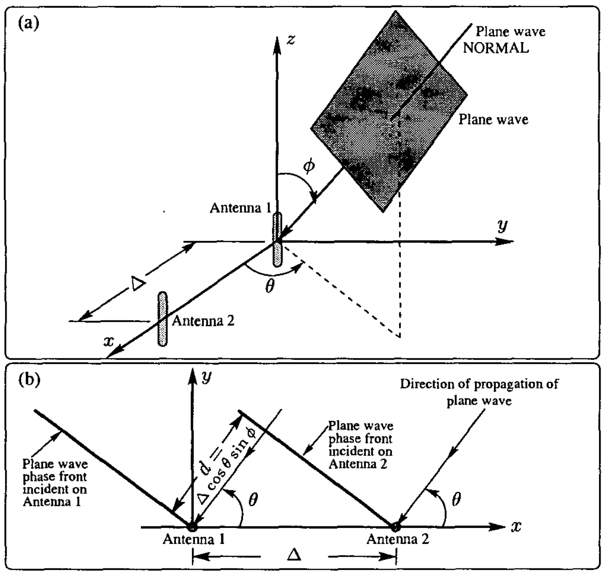

Fig. 1. (a) Antenna geometry used to determine the Direction-Of-Arrival (DOA) of a plane wave incident on a two element linear array oriented along x-axis. (b) Baseband model of a linear array oriented along the x-axis receiving a plane wave from direction $\{\theta, \phi\}$.

$$d = \Delta \cos\theta \sin\phi \tag{5}$$

where Δ is inter-element spacing, θ is the angle between the projection of the plane-wave normal onto the x-y plane and the x-axis, ϕ is the angle between the plane-wave normal and the z-axis. The angle tuple $\{\theta, \phi\}$ is called the Direction-Of-Arrival (DOA). Then the phase difference, $\psi(\Delta, \theta, \phi, \lambda_o)$, between the signal component on array element antenna two and the reference antenna one is

$$\psi = \frac{2\pi d}{\lambda_o} = \frac{2\pi\Delta\cos\theta\sin\phi}{\lambda_o} = \frac{2\pi f_o \Delta\cos\theta\sin\phi}{c}, \tag{6}$$

where $\lambda_o = c/f_o$ is the transmission wavelength, c is the speed of the light, 3×10^8 m/s, f_o is the transmission frequency in *Hz*. Most often the inter-element spacing Δ is taken to be multiple of wavelength λ_o, that is

$$\Delta = m\lambda_o, \quad m \in \mathbb{R}^+. \tag{7}$$

Then Equation (6) can be written as

$$\psi = 2\pi m \cos\theta \sin\phi \tag{8}$$

where m is expressed in terms of wavelengths.

Without loss of generality, we let the antenna number one, with the physical channel complex gains coefficients $\mathbf{c}_1 = [c_{1,-K}, \cdots, c_{1,0}, \cdots, c_{1,L}]^T$, to be the reference antenna. Then the complex valued gains $c_{i,k}$ for remaining antennae can be given by

$$c_{i,k} = u_i(\theta_k, \phi_k) c_{1,k}, \quad 2 \leq i \leq N_A, -K \leq k \leq L, \tag{9}$$

where $u_i(\theta_k, \phi_k) = e^{-j(i-1)\psi(\theta_k,\phi_k)}$. Substituting Equation (9) into (3) the composite channel impulse responses for the remaining antennae are written as

$$h_i(t) = \sum_{k=-K}^{L} u_i(\theta_k, \phi_k) c_{1,k} p(t - \tau_k), \; 2 \leq i \leq N_A, \tag{10}$$

where in the case of a *uniform linear array* of N_A identical antennas, with identical inter-element spacing Δ, and the plane-wave assumption holds, the *array manifold*

$$\mathbf{u}(\theta_k, \phi_k) = [u_1(\theta_k, \phi_k), u_2(\theta_k, \phi_k), \cdots, u_{N_A}(\theta_k, \phi_k)]^T$$

has the form [4]

$$\mathbf{u}(\theta_k, \phi_k) = [1, e^{-j\psi(\theta_k,\phi_k)}, \cdots, e^{-j(N_A-1)\psi(\theta_k,\phi_k)}]^T \tag{11}$$

where $\psi(\theta_k, \phi_k) = 2\pi m \cos\theta_k \sin\phi_k$ and $m \in \mathbb{R}^+$ is specified in terms of wavelengths. Combining Equations (1) and (10) the received signal at the output of receive filter, $\mathbf{y}[n] = [y_1[n], y_2[n], \cdots, y_{N_A}[n]]^T$, in vector form is given by

$$\mathbf{y}[n] = \left(\sum_k \sum_{l=-K}^{L} I_k \mathbf{u}(\theta_l, \phi_l) c_{1,l} p(t - \tau_l) \right) \Bigg|_{t=nT} + \tilde{\boldsymbol{\nu}}[n], \tag{12}$$

where $\tilde{\boldsymbol{\nu}}[n] = [\tilde{\nu}_1[n], \tilde{\nu}_2[n], \cdots, \tilde{\nu}_{N_A}[n]]^T$.

1.2. Review of Least-Squares Channel Estimation

Without loss of generality symbol rate sampled composite CIR's for each antenna $h_i[n]$ can be written as a finite dimensional vector

$$\mathbf{h}_i = [h_i[-N_a], \cdots, h_i[-1], h_i[0], h_i[1], \cdots, h_i[N_c]]^T \tag{13}$$

where N_a and N_c denote the number of anti-causal and the causal taps of the channel, respectively, and $N_a + N_c + 1$ is the total memory of the channel. Then we concatenate these N_A CIR vectors to get

$$\mathbf{h} = [\mathbf{h}_1, \cdots, \mathbf{h}_{N_A}]^T, \tag{14}$$

where $\mathbf{h}$ is of dimension $N_A(N_a + N_c + 1) \times 1$. Based on Equation (1) and assuming that $N \geq N_a + N_c + 1$, the pulse matched filter output corresponding only to the known training symbols for ith antenna is given by the vector $\mathbf{y}_i = [y_i[N_c], y_i[N_c + 1], \cdots, y_i[N-1-N_a]]^T$. similarly the filtered noise vector per antenna is denoted by $\tilde{\boldsymbol{\nu}}_i = [\nu_i[N_c], \nu_i[N_c+1], \cdots, \nu_i[N-1-N_a]]^T$. By concatenating the N_A antenna output and noise vectors respectively as $\mathbf{y} = [\mathbf{y}_1, \cdots, \mathbf{y}_{N_A}]^T$, and $\tilde{\boldsymbol{\nu}} = [\tilde{\boldsymbol{\nu}}_1, \cdots, \tilde{\boldsymbol{\nu}}_{N_A}]^T$, we obtain

$$\mathbf{y} = \mathbf{A}\mathbf{h} + \tilde{\boldsymbol{\nu}}, \tag{15}$$

$$\mathbf{A} = \mathbf{I}_{N_A} \otimes \tilde{\mathbf{A}}, \tag{16}$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix} a_{N_c+N_a} & a_{N_c+N_a-1} & \cdots & a_0 \\ a_{N_c+N_a+1} & a_{N_c+N_a} & \cdots & a_1 \\ \vdots & \vdots & \ddots & \vdots \\ a_{N-1} & a_{N-2} & \cdots & a_{N-1-N_a-N_c} \end{bmatrix}, \quad (17)$$

where $\tilde{\mathbf{A}}$ is $(N-N_a-N_c)\times(N_a+N_c+1)$ dimensional *convolution matrix* composed only of the known training symbols, $\mathbf{I}_{N_A}$ is N_A dimensional identity matrix, $\otimes$ denotes the Kronecker matrix product. As long as the matrix $\mathbf{A}$ is a tall matrix and of full column rank, that is

(i) $N \geq 2(N_a+N_c)+1$,

(ii) $\text{rank}\{\mathbf{A}\} = N_A(N_a+N_c+1)$

then the least squares solution which minimizes the objective function $J_{LS}(\mathbf{h}) = \|\mathbf{y}-\mathbf{Ah}\|^2$ exists and unique, and is given by $\widehat{\mathbf{h}}_{LS} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{Y}$.

For a single antenna receiver the problems associated with the standard least squares based CIR estimation, with an insufficient number of training symbols, is summarized by Özen, et al[3].

2. OVERVIEW OF THE PROPOSED CIR ESTIMATOR

The channel estimation is performed in two steps using symbol-spaced received samples after the receiver pulse matched filter. In the first step, the received samples are correlated with the stored training sequence and thresholding is applied in order to determine the locations of the multipath delays. The purpose of the second step is to incorporate the transmitted pulse shape $p(t)$ into the channel impulse response. To do this, we locate three copies of $p(t)$ shifted by one-half of a symbol period around each multipath location and estimate complex scaling factors using a modified least squares approach.

We first obtain a raw channel estimate in order to located the TOA's by cross-correlating a_n (known and stored at the receiver) with the received sequence $y_i[n]$ at ith antenna output. The initial (uncleaned) impulse response estimate for ith antenna, $\tilde{h}_{\{u,i\}}[n]$, is determined by [2]

$$\tilde{h}_{\{u,i\}}[n] = \frac{1}{r_a[0]}\sum_{k=0}^{N-1} a_k^* y_i[k+n], \quad (18)$$

for $n = -N_a, \cdots, 0, \cdots, N_c$ where $r_a[0] = \sum_{k=0}^{N-1}\|a_k\|^2$. Equivalently Equation (18) can be written as

$$\tilde{\mathbf{h}}_{\{u,i\}} = \frac{1}{r_a[0]}\mathbf{A}_t^H \tilde{\mathbf{y}}_i, \quad (19)$$

where $\mathbf{A}_t$ is a $(N+N_a+N_c)\times(N_a+N_c+1)$ Toeplitz matrix with first column $[a_0, a_1, \cdots, a_{N-1}, 0, \cdots, 0]^T$, and first row $[a_0, 0, \cdots, 0]$, $\tilde{\mathbf{y}} = [y_i[-N_a], \cdots, y_i[N-N_c-1]]^T$. In order to get rid of the sidelobes of the aperiodic autocorrelation we can simply invert the normalized autocorrelation matrix $\mathbf{R}_{aa}$ of the training symbols, defined by

$$\mathbf{R}_{aa} = \frac{1}{r_a[0]}\mathbf{A}_t^H\mathbf{A}_t. \quad (20)$$

Then the *cleaned* channel estimate $\tilde{\mathbf{h}}_{\{c,i\}}$ is obtained from

$$\tilde{\mathbf{h}}_{\{c,i\}} = \mathbf{R}_{aa}^{-1}\tilde{\mathbf{h}}_{\{u,i\}}, \quad (21)$$

however the channel estimate $\tilde{\mathbf{h}}_{\{c,i\}}$ obtained by Equation (21) has the contributions due to unknown symbols prior to and after the training sequence, as well as the additive channel noise; only the sidelobes due to aperiodic auto-correlation is removed. The CIR obtained by Equation (21) will ideally yield *peaks* at $\{D_k^a \in \mathbb{Z}^+\}$, for $k = -K, \cdots, -1, 0$, and at $\{D_k^c \in \mathbb{Z}^+\}$, for $k = 1, \cdots, L$. These delays are the sampling instants closest to the locations of the actual physical channel multi-path TOAs τ_k, $k = -K, \cdots, -1, 0, 1, \cdots, L$, within a symbol interval. The relationship between the actual TOA's and and the estimated TOAs is given by

$$D_k^a = -\text{round}\left(\frac{\tau_k}{T}\right), \quad \text{for } -K \leq k \leq 0, \quad (22)$$

$$D_k^c = \text{round}\left(\frac{\tau_k}{T}\right), \quad \text{for } 1 \leq k \leq L. \quad (23)$$

Assuming that there is a *level thresholding* algorithm taking place right after the cross-correlations, which is in the form of setting the estimated channel taps to zero if they are below a certain preselected threshold; that is

$$\text{set} \quad \widehat{h}_{\{th,i\}}[n] = \begin{cases} 0, & \text{if } \|\tilde{h}_{\{c,i\}}[n]\| < \varepsilon \\ \tilde{h}_{\{c,i\}}[n], & \text{otherwise,} \end{cases} \quad (24)$$

for $n = -N_a, \cdots, -1, 0, 1, \cdots, N_c$, then in general we can choose the tap location with largest magnitude for *all* antennae and denote it as the *cursor* (reference) path, and the TOAs prior to and after this reference TOA are denoted as pre- and post-cursor channel impulse responses on all antenna outputs. Without loss of generality we assume that this largest spike occurs at the first antenna output which lets us denote its complex gain by $\tilde{\alpha}_{1,0}$. Then we write the estimated transfer function *after cleaning* and *thresholding* takes place as

$$\widehat{H}_{\{th,i\}}(z) = \tilde{\alpha}_{i,0}\Big(\beta_{i,K}z^{D_K^a} + \cdots + \beta_{i,1}z^{D_1^a} + 1 + \alpha_{i,1}z^{-D_1^c} + \cdots + \alpha_{i,L}z^{-D_L^c}\Big) \quad (25)$$

where $\beta_{i,k}$'s and $\alpha_{i,k}$'s are the complex gains for ith channel and D_k^a's, D_k^c's are the delays of the estimated channel corresponding to the *pre-cursor* (anti-causal) part, and the *post-cursor* (causal) part respectively. It is assumed that $1 \leq D_1^c < \cdots < D_L^c$, and similarly $1 \leq D_1^a < \cdots < D_K^a$, and we also have $D_K^a + N_q = N_a$ and $D_L^c + N_q = N_c$.

After the thresholding takes place we will end up only with the peaks of the channel taps, which implies the fact that the tails of the pulse shape that are buried under the "noisy" correlation output may end up being zeroed out entirely. Now we will show how to recover the pulse shape $p(t)$ into the CIR estimate. The idea is that for each multi-path we would like to approximate the shifted and scaled copy of the pulse shape $p(t)$ (shifted by τ_k and scaled by $c_{i,k}$) by a linear combination of three pulse shape functions equi-spaced by half a symbol interval centered at the closest symbol time. More precisely

$$c_{i,k}p(nT-\tau_k) \approx \begin{cases} \sum_{l=-1}^{1}\gamma_l^{(i,k)}p((n+D_k^a-\frac{l}{2})T), & -K \leq k \leq 0 \\ \sum_{l=-1}^{1}\gamma_l^{(i,k)}p((n-D_k^c-\frac{l}{2})T), & 1 \leq k \leq L \end{cases} \quad (26)$$

where $\{\gamma_l^{(i,k)}, -K \leq k \leq L\}_{l=-1}^{1} \subset \mathbb{C}^1$ for $1 \leq i \leq N_A$. By making this approximation we can also efficiently recover the tails of the complex pulse shape $p(t)$ which are generally buried under the "noisy" output of the correlation processing, and are completely lost when thresholding is applied. To accomplish this approximation we introduce three vectors $\mathbf{p}_k$, for $k = -1, 0, 1$, each containing T spaced samples of the complex pulse shape $p(t)$ shifted by $kT/2$, such that

$$\mathbf{p}_k = [p(-N_qT-\frac{kT}{2}),\cdots,p(-\frac{kT}{2}),\cdots,p(N_qT-\frac{kT}{2})]^T, \quad (27)$$

for $k = -1, 0, 1$, and by concatenating these vectors side by side we define a $(2N_q + 1) \times 3$ matrix $\mathbf{P}$ by

$$\mathbf{P} \quad = \quad [\mathbf{p}_{-1}, \ \mathbf{p}_0, \ \mathbf{p}_1]. \quad (28)$$

Then we form the matrix denoted by $\tilde{\Gamma}$ whose columns are composed of the shifted vectors $\mathbf{p}_k$, where the shifts represent the relative delays of the multi-paths; that is

$$\tilde{\Gamma} = \begin{bmatrix} \mathbf{P} & & \\ \mathbf{0}_{(D_K^a + D_L^c)\times 3} & \ddots \quad \mathbf{0}_{D_K^a \times 3} & \\ & \mathbf{P} & \\ & \mathbf{0}_{D_L^c \times 3} & \ddots \quad \mathbf{0}_{(D_K^a+D_L^c)\times 3} \\ & & \mathbf{P} \end{bmatrix}, \quad (29)$$

where $\tilde{\Gamma}$ is of dimension $(D_K^a + D_L^c + 2N_q + 1) \times 3(K+L+1)$, and $\mathbf{0}_{m\times n}$ denotes an m by n zero matrix. Then the observation vector $\mathbf{y}$, and the convolution matrix, $\mathbf{A}$, composed only of the known training symbols are defined as in Equations (15, 16) respectively. Since it was assumed that $q(t)$ spans N_q symbol durations, it implies that $q[n]$ has N_q+1 sample points, which in turn implies $p[n]$ has $2N_q+1$ samples. We define $\boldsymbol{\gamma}^{(i,k)} = [\gamma_{-1}^{(i,k)}, \gamma_0^{(i,k)}, \gamma_1^{(i,k)}]$ for $-K \leq k \leq L$, $\boldsymbol{\gamma}^{(i)} = [\boldsymbol{\gamma}^{(i,-K)}, \cdots, \boldsymbol{\gamma}^{(i,0)}, \cdots, \boldsymbol{\gamma}^{(i,L)}]^T$, as the unknown vector of the coefficients for ith antenna $\{\gamma_n^{(i,k)}, n = -1, 0, 1; i = 1, \cdots, N_A; k = -K, \cdots, 0, \cdots, L\}$, of length $3(K + L + 1)$. Then the vector that contains all the coefficients $\{\gamma_l^{(i,k)}\}$ is $\boldsymbol{\gamma} = [\boldsymbol{\gamma}^{(1)}, \cdots, \boldsymbol{\gamma}^{(N_A)}]^T$. Then we can write the observation vector as

$$\mathbf{y} \quad = \quad \mathbf{A}\Gamma\boldsymbol{\gamma} + \tilde{\boldsymbol{\nu}} \quad (30)$$

where $\Gamma = \mathbf{I}_{N_A} \otimes \tilde{\Gamma}$, and $\tilde{\boldsymbol{\nu}}$ is the observation noise vector. Using the least squares arguments again, we can estimate the unknown coefficient vector $\boldsymbol{\gamma}$ as

$$\widehat{\boldsymbol{\gamma}}_{BLS} \quad = \quad \left(\Gamma^H \mathbf{A}^H \mathbf{A}\Gamma\right)^{-1} \Gamma^H \mathbf{A}^H \mathbf{y}. \quad (31)$$

Once the vector $\widehat{\boldsymbol{\gamma}}_{BLS}$ is obtained, the new channel estimate $\widehat{\mathbf{h}}_{BLS}$, where the pulse tails are recovered back, can be obtained by

$$\widehat{\mathbf{h}}_{BLS} \quad = \quad \Gamma\widehat{\boldsymbol{\gamma}}_{BLS}. \quad (32)$$

The subscript $\{.\}_{BLS}$ stands for the *Blended Least Squares* which is motivated by the structure of the algorithm that is based on blending the TOA estimates, obtained by correlation, cleaning, an thresholding, into the least squares channel estimation.

2.1. Channel Noise Variance Estimation

Once we define the estimated channel vector as in Equation (32) we can similarly define the reconstructed observation vector $\widehat{\mathbf{y}}_i$, based on $\widehat{\mathbf{h}}_{BLS} = [\widehat{\mathbf{h}}_{BLS,1}, \cdots, \widehat{\mathbf{h}}_{BLS,2}]^T$, by

$$\widehat{\mathbf{y}}_i = \tilde{\mathbf{A}}\widehat{\mathbf{h}}_{BLS,i} = \tilde{\mathbf{A}}\tilde{\Gamma}\left(\tilde{\Gamma}^H \tilde{\mathbf{A}}^H \tilde{\mathbf{A}}\tilde{\Gamma}\right)^{-1} \tilde{\Gamma}^H \tilde{\mathbf{A}}^H \mathbf{y}_i. \quad (33)$$

Then the variance of the colored noise samples at the output of the matched filter $\sigma_{\tilde{\nu}_i}^2$, for ith antenna can be estimated using the estimator

$$\widehat{\sigma^2}_{\tilde{\nu}_i} \quad = \quad \frac{1}{2(N - N_a - N_c)} \|\widehat{\mathbf{y}}_i - \mathbf{y}_i\|^2. \quad (34)$$

Note that $(N - N_a - N_c)$ is the length of the observation vector $\mathbf{y}_i$. Then the variance of the white noise prior to pulse matched filter is computed from

$$\widehat{\sigma^2}_{\nu_i} \quad = \quad N_{0,i} = \frac{\widehat{\sigma^2}_{\tilde{\nu}_i}}{E_q} \quad (35)$$

where E_q is the energy of the pulse matched filter.

3. SIMULATIONS OF THE PROPOSED ALGORITHM

We considered an 8-VSB [1] receiver with $N_A = 2$ antenna, with inter-element spacing of one wavelength. 8-VSB system has a complex raised cosine pulse shape [1]. The CIR corresponding to first antenna has 5 multipaths (4 ghosts and 1 main path):

$$c_1(t) = (0.45+j0.2)\delta(t+50.2T)+\delta(t)-0.5\delta(t-20.12T)+$$
$$(0.4-j0.5)\delta(t-11.3T)+(0.22-j0.39)\delta(t-385.1T). \quad (36)$$

The DOAs for each multipath are $\boldsymbol{\theta} = [5\pi/12, 5.5\pi/12, 4\pi/12, 11\pi/18, 10\pi/18]$, and $\phi = \pi/2$ for all multipaths. The CIR for antenna 2 is calculated from Equation (9). The Signal-to-Noise-Ratio (SNR) per antenna is 20 dB at the input to the receive pulse matched filter, and it is calculated for i'th antenna by

$$\text{SNR}_i = E_s \left\|(c_i(t) * q(t))|_{t=nT}\right\|^2 / N_{0,i}, \quad (37)$$

where $E_s = 21$ for 8-VSB system, and $N_{0,i}$ is the channel noise variance. Figure 2 parts (a-d) show the actual CIR's, and Figure 2 parts (e-h) show the estimated CIRs where the pulse-shape is recovered back into the channel estimate. We must note the channel in (36) has a *delay spread* of more than 435 symbol durations which is too long for the standard least squares algorithm to handle[3].

4. REFERENCES

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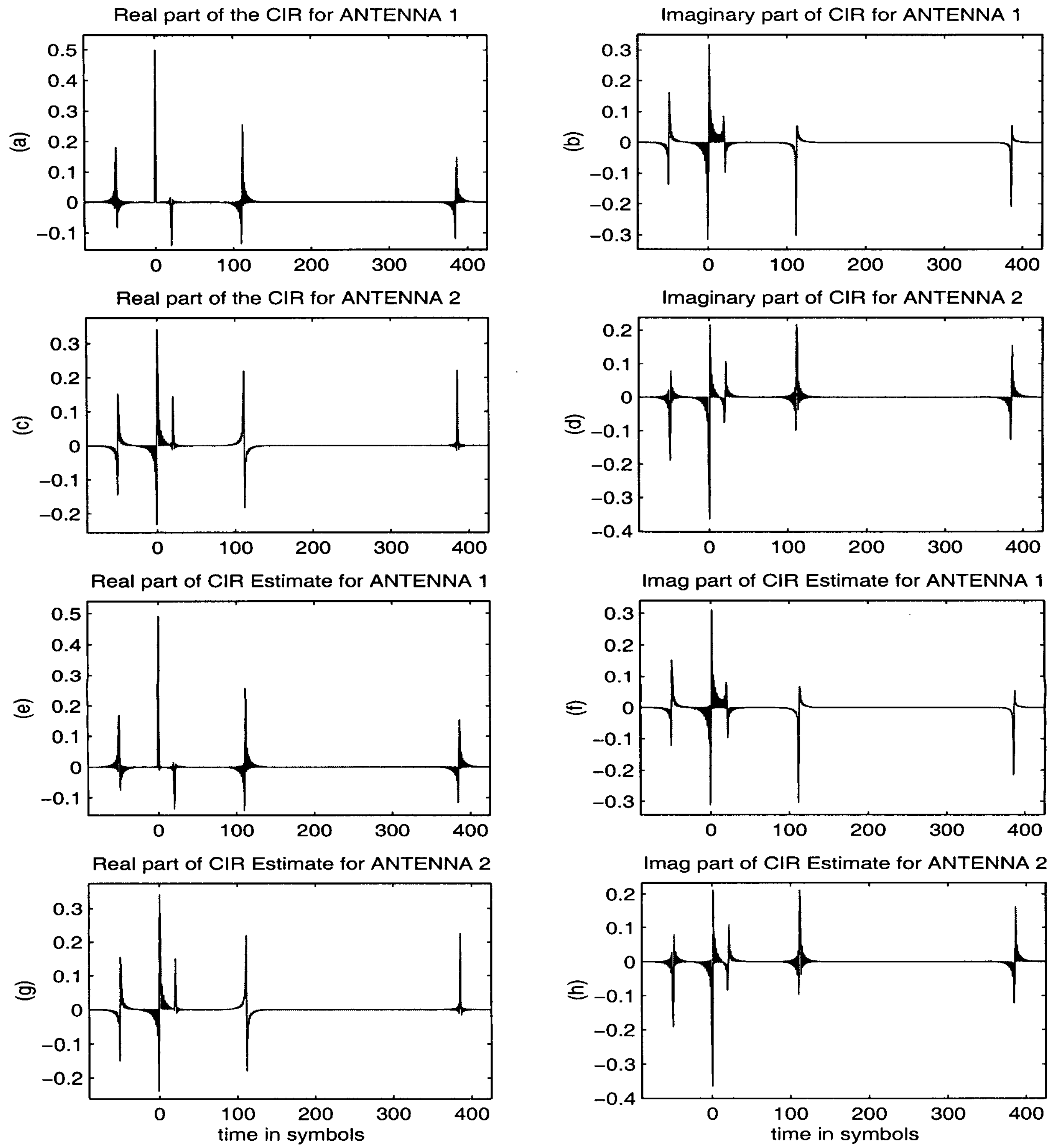

Fig. 2. (a) and (b) show the real and imaginary parts of the channel impulse response seen by the first antenna; (c) and (d) show the real and imaginary parts of the channel impulse response seen by the second antenna; (e) and (f) show the real and imaginary parts of the estimated CIR for the first antenna; (g) and (h) show the real and imaginary parts of the estimated CIR for the second antenna .